K24U 0928
Reg. No. :
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# IV Semester B.Sc. Degree (C.B.C.S.S. - Supplementary/One Time Mercy Chance) Examination, April 2024 (2014 to 2018 Admissions) COMPLEMENTARY COURSE IN MATHEMATICS 4C04 MAT-PH : Mathematics For Physics and Electronics - IV 

Time : 3 Hours
Max. Marks : 40

## SECTION - A

All the first 4 questions are compulsory. They carry 1 mark each.

1. What curve is given by the parametric representation $r(t)=[3 \cos t, 4 \sin t, t]$.
2. Find the gradient of $f(x, y)=x^{2}-y^{2}$ at $(-1,3)$.
3. Let $F=\left[y^{2},-x^{2}\right]$ and $C$ be the straight line segment from $(0,0)$ to $(1,4)$. Find $\int_{C} F(r) \cdot d r$.
4. State Newton's forward difference interpolation formula.

## SECTION - B

Answer any 7 questions from among the questions 5 to 13 . These questions carry 2 marks each.
5. Prove that $\nabla(\mathrm{fg})=\mathrm{f} \nabla \mathrm{g}+\mathrm{g} \nabla \mathrm{f}$.
6. Prove that $\mathrm{v}=[2 \mathrm{y}, 2 \mathrm{z}, 4 \mathrm{x}+\mathrm{z}]$ is irrotational.
7. Find the value of $\int_{C} F(r) \cdot d r$, when $F(r)=z i+x j+y k$ and $C$ is $r(t)=\cos t i+\sin t j+3 t k$.
8. Using Green's theorem, find the area of the ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$.
9. Find a real root of the equation $x^{3}-x-1=0$, using bisection method.
10. Certain corresponding values of $x$ and $\log _{10} x$ are (300, 2.4771), (304, 2.4829), $(305,2.4843)$ and (307, 2.4871). Find $\log _{10} 301$, using Lagrange's interpolation formula.
11. From the following table evaluate $\int_{7.47}^{7.52} f(x) d x$, using Trapezoidal rule.

| $\mathbf{x}$ | 7.47 | 7.48 | 7.49 | 7.50 | 7.51 | 7.52 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{f}(\mathbf{x})$ | 1.93 | 1.95 | 1.98 | 2.01 | 2.03 | 2.06 |

12. From the Taylor series for $y(x)$, find $y(0.1)$ correct to four decimal places if $y(x)$ satisfies $y^{\prime}=x-y^{2}$ and $y(0)=1$.
13. Use Picard's method to obtain a series solution for $\frac{d y}{d x}=1+x y, y(0)=1$.
SECTION - C

Answer any 4 questions from among the questions 14 to 19. These questions carry 3 marks each.
( $4 \times 3=12$ )
14. Find the tangent to the ellipse $\frac{x^{2}}{4}+y^{2}=1$ at $P\left(\sqrt{2}, \frac{1}{\sqrt{2}}\right)$.
15. Find the length of the catenary $r(t)=t i+\cosh t j$ from $t=0$ to $t=1$.
16. Using divergence theorem, evaluate $I=\iint_{S}\left(x^{3} d y d z+x^{2} y d z d x+x^{2} z d x d y\right)$ where $S$ is the closed surface consisting of the cylinder $x^{2}+y^{2}=a^{2},(0 \leq z \leq b)$ and the circular disks $\mathrm{z}=0$ and $\mathrm{z}=\mathrm{b}$.
17. Find the cubic polynomial which takes the following values: $y(1)=24$, $y(3)=120, y(5)=336$ and $y(7)=720$.
18. From the following values of $x$ and $y$ obtain $\frac{d y}{d x}$ at $x=1.2$.

| $\mathbf{x}$ | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 | 2.2 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{y}$ | 2.7183 | 3.3201 | 4.0552 | 4.9530 | 6.0496 | 7.3891 | 9.0250 |

19. Using Euler's modified method, find an approximate value of $y(x)$ when $x=0.4$, given that $\frac{d y}{d x}=x+y, y(0)=0$ by choosing $h=0.2$.
SECTION-D

Answer any 2 questions from among the questions 20 to 23. These questions carry 5 marks each.
20. Find the torsion of the curve $r(t)=[a \cos t, a \sin t, c t]$.
21. Verify Stoke's theorem for the function $F=y i+z j+z k$ integrated round the paraboloid $z=1-\left(x^{2}+y^{2}\right), z \geq 0$.
22. A solid of revolution is formed by rotating about the $x$-axis the area between the $x$-axis, the lines $x=0$ and $x=1$ and a curve through the points with the following coordinates:

| $\mathbf{x}$ | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{y}$ | 1.0000 | 0.9896 | 0.9589 | 0.9089 | 0.8415 |

Estimate the volume of the solid formed, giving the answer to three decimal places.
23. Use Runge-Kutta fourth order formula to find $y(0.2)$ and $y(0.4)$ given that $y^{\prime}=\frac{y^{2}-x^{2}}{y^{2}+x^{2}}, y(0)=1$.

