K24U 0731

Reg. No. : ....

Name : ....

IV Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/ Improvement) Examination, April 2024
(2019 to 2022 Admissions)
COMPLEMENTARY ELECTIVE COURSE IN MATHEMATICS
4C04 MAT – PH : Mathematics for Physics – IV

Time : 3 Hours Max. Marks : 40

PART – A

Answer **any four** questions from this Part. **Each** question carries **1** mark.

1. Define a linear partial differential equation.
2. Define a gradient field of a differential function f(x, y, z).
3. Give an example of an non-orientable surface.
4. State trapezoidal rule.
5. State the fundamental theorem of line integrals. **(4×1=4)**

PART – B

Answer **any seven** questions. **Each** question carries **2** marks.

6. Verify that $u = e^x \cos y, e^x \sin y$ is a solution of the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.
7. Verify that u = v(x) + w(y) with any v and w satisfies the partial differential equation $u_{xy} = 0$.
8. Integrate $f(x, y) = \frac{x^3}{y}$ over the curve $C : y = x^2/2, 0 \le x \le 2$.
9. Prove that curl grad $f = \overline{0}$.
10. State Green's theorem.

P.T.O.

11. Find a parametrization of the sphere $x^2 + y^2 + z^2 = a^2$.

12. Solve $y' = -y$ with the condition that $y(0) = 1$ by Euler's method.

13. Explain the method of solution of differential equation $y'(x, y) = f(x, y)$ with the initial condition $y(x_0) = y_0$ by Taylor series.

14. Explain Euler's method to find the solution of the differential equation.

15. Find the divergence of the vector field $\bar{F}(x, y, z) = z\hat{j}$.

16. Find the work done by the force field $\bar{F} = x\hat{i} + y\hat{j} + z\hat{k}$ in moving an object along the curve C parametrized by $\bar{r}(t) = \cos(\pi t)\hat{i} + t^2\hat{j} + \sin(\pi t)\hat{k},\ 0 \le t \le 1$. **(7×2=14)**

PART – C

Answer **any four** questions. **Each** question carries **3** marks.

17. Solve $u_{xx} + 2u_{xy} + u_{yy} = 0$.

18. If u_1 and u_2 are solutions of $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ in some region R. Prove that $u = c_1u_1 + c_2u_2$ is also a solution of the above partial differential equation.

19. Find the flux of $\bar{F} = (x - y)\hat{i} + x\hat{j}$ across the circle $x^2 + y^2 = 1$ in the xy-plane.

20. Integrate $G(x, y, z) = \sqrt{1 - x^2 - y^2}$ over the "football" surface S formed by rotating the curves $x = \cos z$, $y = 0$, $-\frac{\pi}{2} \le z \le \frac{\pi}{2}$ around the z-axis.

21. A solid of revolution is formed by rotating about the x-axis the area between the x-axis, the lines x = 0 and x = 1, and a curve through the points with the following coordinates :

X	Y
0.00	1.0000
0.25	0.9896
0.50	0.9589
0.75	0.9089
1.00	0.8415

22. Evaluate $y = \int_0^{\frac{\pi}{2}} \sqrt{\sin x}\, dx$ using Simpson's $\frac{1}{3}$ rule with $h = \frac{\pi}{12}$.

23. Find the surface area of a sphere of radius a. **(4×3=12)**

PART – D

Answer **any two** questions. **Each** question carries **5** marks.

24. Find the solution of the one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, subject to the initial conditions $u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$, $(0 \le x \le L)$.

25. Verify both forms of Green's theorem for the vector field $\bar{F}(x, y) = (x - y)\hat{i} + x\hat{j}$ and the region R bounded by the unit circle $C : \bar{r}(t) = \cos t\,\hat{i} + \sin t\,\hat{j},\ 0 \le t \le 2\pi$.

26. Given $\frac{dy}{dx} = y - x$, where $y(0) = 2$, find $y(0.1)$ and $y(0.2)$ correct to four decimal places.

27. Find the outward flux of the field $\bar{F} = (y - x)\hat{i} + (z - y)\hat{j} + (y - x)\hat{k}$ across the boundary of the cube D bounded by the planes $x = \pm 1$, $y = \pm 1$ and $z = \pm 1$. **(2×5=10)**