K22U 1562

Reg. No. : ....

Name : ....

IV Semester B.Sc. Degree CBCSS (OBE) Regular/Supplementary/ Improvement Examination, April 2022
(2019 Admission Onwards)
COMPLEMENTARY ELECTIVE COURSE IN MATHEMATICS
4C04MAT-PH : Mathematics for Physics – IV

Time : 3 Hours

Max. Marks : 40

PART – A

Answer **any four** questions from this Part. **Each** question carries **1** mark.

1. Find the order of the partial differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$.
2. Find the gradient of $f(x, y, z) = \sqrt{\ln(x^2 + y^2 + z^2)}$.
3. Write a parametrization of the sphere $x^2 + y^2 + z^2 = a^2$.
4. Find the divergence of the vector field $F(x, y) = \frac{-y}{x^2 + y^2}\hat{i} + \frac{x}{x^2 + y^2}\hat{j}$.
5. Define Trapezoidal rule.

(4×1=4)

PART – B

Answer **any seven** questions from this Part. **Each** question carries **2** marks.

6. Show that $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ is a parabolic partial differential equation.
7. Solve the partial differential equation $u_{yy} = 0$.
8. Find the line integral of $f(x, y) = x - y + 3$ along the curve $r(t) = \cos t\,\hat{i} + \sin t\,\hat{j}$, $0 \le t \le 2\pi$.

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9. What is the circulation density of $F = \tan^{-1}\left(\frac{y}{x}\right)\hat{i} + \ln(x^2 + y^2)\hat{j}$?

10. Show that sin y cos x dx + cos y sin x dy + dz is exact.

11. Integrate $G(x, y, z) = x^2$ over the unit sphere $x^2 + y^2 + z^2 = 1$.

12. Find the curl of $F = (x - y)\hat{i} + (y - z)\hat{j} + (z - x)\hat{k}$.

13. Prove that $\nabla \times \nabla f = 0$.

14. Use Trapezoidal rule with n = 4 to approximate $\int_0^2 x^3 dx$.

15. Find an upper bound for the error in estimating $\int_0^2 5x^4\, dx$ using Simpson's $\frac{1}{3}$ - rule with n = 4.

16. Find the Taylor series for y(x) if $\frac{dy}{dx} = 1 + xy$ and y(0) = 1. **(7×2=14)**

PART – C

Answer **any four** questions from this Part. **Each** question carries **3** marks.

17. Transform the partial differential equation $u_{xx} + 2u_{xy} + u_{yy} = 0$ into a normal form.

18. If u_1 and u_2 are solutions of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in some region R, then prove that $u = c_1 u_1 + c_2 u_2$ where c_1 and c_2 are constants is also a solution of the above partial differential equation.

19. Find the circulation of the field $F = (x - y)\hat{i} + x\hat{j}$ around the circle $r(t) = \cos t\,\hat{i} + \sin t\,\hat{j}$, $0 \le t \le 2\pi$.

20. Show that $F = (e^x \cos y + yz)\hat{i} + (xz - e^x \sin y)\hat{j} + (xy + z)\hat{k}$ is conservative over its natural domain and find a potential function for it.

21. Find the area of the surface cut from the paraboloid $x^2 + y^2 - z = 0$ by the plane $z = 2$.

22. Use Euler method to find the value of Estimate the value of y when $x = 0.1$ if $y' = x^2 + y$ and $y(0) = 1$.

23. If $\frac{dy}{dx} = y - x$ and $y(0) = 2$, find $y(0.1)$ correct to four decimal places. **(4×3=12)**

PART – D

Answer **any two** questions from this Part. **Each** question carries **5** marks.

24. Find the solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ with boundary condition $u(0, t) = u(L, t) = 0$, $t \geq 0$ and initial condition $u(x, 0) = f(x)$, $u_t(x, 0) = 0$, $0 \leq x \leq L$ where $f(x) = \begin{cases} \frac{2kx}{L} & \text{if } 0 \leq x < L/2 \\ \frac{2K(L-x)}{L} & \text{if } L/2 \leq x \leq L \end{cases}$.

25. Find the counter clockwise circulations and outward flux of the field $F = xy\hat{i} + y^2\hat{j}$ around and over the boundary of the region enclosed by the curves $y = x^2$ and $y = x$ in the first quadrant.

26. Calculate the circulation of the field $F = 2y\hat{i} + 3x\hat{j} - z^2\hat{k}$ around the circle $x^2 + y^2 = 9$ in the XY plane counter clockwise when viewed from above.

27. If $y = A + Bx + Cx^2$ and y_0, y_1, y_2 are the values y corresponding to $x = 0$, h and 2h respectively, prove that $\int_0^{2h} y\,dx = \frac{h}{3}(y_0 + 4y_1 + y_2)$. **(2×5=10)**