K21U 1126

Reg. No. : ....

Name : ....

IV Semester B.Sc. Degree CBCSS (OBE) Regular Examination, April 2021
(2019 Admission Only)
Complementary Elective Course in Mathematics
4C04MAT-PH : MATHEMATICS FOR PHYSICS – IV

Time : 3 Hours

Max. Marks : 40

PART – A

Answer **any four** questions from this part. **Each** question carries **1** mark.

1. Find the order of the partial differential equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$.
2. Find the gradient of $f(x, y, z) = e^z - \ln(x^2 + y^2)$.
3. Define flux density of a vector field F.
4. What is the flux of a three dimensional vector field F across an oriented surface S in the direction of $\hat{n}$?
5. Describe the fourth order Runge-Kutta formula.

(4×1=4)

PART – B

Answer **any seven** questions from this part. **Each** question carries **2** marks.

6. Show that $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$ is a hyperbolic partial differential equation.
7. Solve the partial differential equation $u_{xx} - 16\pi^2 u = 0$.
8. Find the line integral of $f(x, y, z) = x + y + z$ over the straight line segment of from (1, 2, 3) to (0, –1, 1).
9. Find the work done by the conservative field $F = yz\hat{i} + xz\hat{j} + xy\hat{k} = \nabla f$ where $f(x, y, z) = xyz$ along any smooth curve C joining the point A(–1, 3, 9) to B(1, 6, –4).
10. Show that $F = (e^x \cos y + yz)\hat{i} + (xz - e^x \sin y)\hat{j} + (xy + z)\hat{k}$ is conservative over its natural domain.

P.T.O.

11. Integrate $G(x, y, z) = x$ over the parabolic cylinder $y = x^2$, $0 \le x \le 2$, $0 \le z \le 3$.

12. Find the curl of $F = (x^2 - z)\hat{i} + xe^z\hat{j} + xy\hat{k}$.

13. Prove that $\nabla \times \nabla f = 0$.

14. Use Trapezoidal rule with n = 4 to estimate $\int_1^2 x^2 dx$.

15. Find an upper bound for the error in estimating $\int_0^2 5x^4 dx$ using Simpson's $\frac{1}{3}$ rule with n = 4.

16. Using Euler's method solve $y' = 1 + y^2$, $y(0) = 0$.

(7×2=14)

PART – C

Answer **any four** questions from this part. **Each** question carries **3** marks.

17. Transform the partial differential equation $xu_{xx} - yu_{xy} = 0$ into a normal form.

18. If u_1 and u_2 are solutions of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in some region R, then prove that $u = c_1u_1 + c_2u_2$ where c_1 and c_2 are constants is also a solution of the above partial differential equation.

19. Evaluate the line integral $\int_C -ydx + zdy + 2xdz$ where C is the helix $r(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + t\hat{k}$, $0 \le t \le 2\pi$.

20. Find the surface area of a sphere of radius a.

21. Find the flux $F = xy\hat{i} + yz\hat{j} + xz\hat{k}$ outward through the surface of the cube cut from the first octant of the planes x = 1, y = 1 and z = 1.

22. Estimate the value of the integral $\int_1^3 \frac{1}{x} dx$ by Simpson's rule with 4 strips and determine the error.

23. If $\frac{dy}{dx} = 1 + y^2$ with $y(0) = 0$, find $y(0.2)$ correct to four decimal places.

(4×3=12)

PART – D

Answer **any two** questions from this part. **Each** question carries **5** marks.

24. Find the solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ with boundary condition $u(0, t) = u(L, t) = 0, t \geq 0$ and initial condition $u(x, 0) = f(x), u_t(x, 0) = 0, 0 \leq x \leq L$

    where $f(x) = \begin{cases} \frac{2kx}{L} \text{ if } 0 \leq x < L/2 \\ \frac{2k(L-x)}{L} \text{ if } L/2 \leq x \leq L \end{cases}$

25. Verify both forms of Green's theorem for the vector field $F(x, y, z) = (x - y)\hat{i} + x\hat{j}$ and the region R bounded by the unit circle $C : r(t) = \cos t\ \hat{i} + \sin t\ \hat{j}, 0 \leq t \leq 2\pi$.

26. Verify divergence theorem for the expanding vector field $F = x\hat{i} + y\hat{j} + z\hat{k}$ over the sphere $x^2 + y^2 + z^2 = a^2$.

27. Show that the differential equation $\frac{d^2y}{dx^2} = -xy, y(0) = 1, y'(0) = 1$ has the series solution $y = 1 - \frac{x^3}{3!} + \frac{1 \times 4}{6!}x^6 - \frac{1 \times 4 \times 7}{9!}x^9 + \cdots$.

(2×5=10)
