K20U 0886

Reg. No. : ....

Name : ....

IV Semester B.Sc. Degree (CBCSS-Reg./Sup./Imp.) Examination, April 2020
(2014 Admn. Onwards)
COMPLEMENTARY COURSE IN MATHEMATICS
4C04 MAT-CS : Mathematics For Computer Science – IV

Time : 3 Hours

Max. Marks : 40

SECTION – A

All the **first 4** questions are **compulsory**. They carry **1** mark **each**.

1. Find the first partial derivatives of $\bar{v}$ = [cosx coshy, – sinx sinhy].
2. A line integral is path independent in a domain D if and only if its value around every closed path in D is zero. State True or False.
3. State Stoke's theorem.
4. The percentage error ϵ_r is defined by ϵ_r = ...

(4×1=4)

SECTION – B

Answer **any 7** questions from among the questions **5** to **13**. These questions carry **2** marks **each**.

5. Find the directional derivative of $f(x, y, z) = 2x^2 + 3y^2 + z^2$ at the point (2, 1, 3) in the direction of $\bar{i} - 2\bar{k}$.
6. If $\bar{v} = yz\bar{i} + 3zx\bar{j} + z\bar{k}$, then directly compute div(curl $\bar{v}$).
7. If f(x, y, z) is a twice continuously differentiable scalar function, then show that curl(grad f) = 0.
8. Use Green's theorem to evaluate $\int_C \bar{F}.d\bar{r}$ counterclockwise around the square C whose vertices are (0, 0), (π/2, 0), (π/2, π/2) and (0, π/2) when $\bar{F}$ is the vector [y sin x, 2x cosy].
9. Using the method of false position find an approximate numerical solution of $x^{2.2} = 69$ lying between 5 and 8.

P.T.O.

10. Find the missing term in the following table using the forward difference operator.

x	0	1	2	3	4
y	1	3	9	–	81

11. Evaluate $\int_0^{\pi} t \sin t \, dt$ using trapezoidal rule with $h = \pi/6$.

12. Given that $\frac{dy}{dx} - 1 = xy$ and $y(0) = 1$. Obtain the Taylor series for $y(x)$ and compute $y(0.1)$ correct to four decimal places.

13. From the differential equation $y' = -y$, estimate the value of $y(0.04)$ by Euler's method with a step size of $h = 0.01$, given that $y(0) = 1$. **(7×2=14)**

SECTION – C

Answer **any 4** questions from among the questions **14** to **19**. These questions carry **3** marks **each**.

14. Determine a and b so that $\bar{v} = [2xy + 3yz, x^2 + axz - 4z^2, 3xy + 2byz]$ is irrotational.

15. Show that the differential form under the integral sign is exact and evaluate $\int_{(\pi, \pi/2, 2)}^{(0, \pi, 1)} -z\sin(xz)\, dx + \cos(y)\, dy - x\sin(xz)\, dz$.

16. Compute the flux of water through the parabolic cylinder $S : y = x^2, 0 \le x \le 2, 0 \le z \le 3$, if the velocity vector is $\bar{v} = \bar{F} = [3z^2, 6, 6xz]$.

17. Find the Lagrange interpolating polynomial of degree two approximating the function $y = \ln(x)$ defined by the following table. Hence determine $\ln(2.7)$.

x	2.0	2.5	3
y = ln(x)	0.69315	0.91629	1.09861

18. Evaluate $\int_0^1 \frac{1}{1+x} dx$ correct to three decimal places by Simpson's rule with $h = 0.5, 0.25$ and 0.125 respectively.

19. Using Picard's method solve the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2 + 1}$, $y(0) = 0$ to find the values of y corresponding to $x = 0.25, 0.5$ and 1.0 correct to three decimal places. **(4×3=12)**

SECTION – D

Answer **any 2** questions from among the questions **20** to **23**. These questions carry **5** marks **each**.

20. a) Express the helix $\bar{r}(t) = [a\cos t, a\sin t, ct]$ $(c \neq 0)$ with arc length s as the parameter.

    b) Find the curvature and torsion of the helix in part (a).

21. Verify divergence theorem for the function $\bar{F}(x, y, z) = 7x\bar{i} - z\bar{k}$ over the sphere $x^2 + y^2 + z^2 = 4$.

22. Compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 1.05 from the following data.

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
y	1.000	1.025	1.049	1.072	1.095	1.118	1.140

23. Use the Runge-Kutta method to solve $10\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$ for the interval $0 < x \leq 0.3$ with $h = 0.1$.

(2×5=10)