K20U 3321

Reg. No. : ....

Name : ....

I Semester B.Sc. Degree CBCSS (OBE) Reg./Sup./Imp. Examination, November 2020
(2019 Admn. Onwards)
COMPLEMENTARY ELECTIVE COURSE IN MATHEMATICS
1C01 MAT – PH : Mathematics for Physics – I

Time : 3 Hours

Max. Marks : 40

PART – A
Short Answer

Answer **any four** questions out of five questions. **Each** question carries **1** mark :

1. Find the derivative of $2x^5 - x^3 - x$.
2. *Write the Maclaurin's series of* $\sin\theta$.
3. Find the rank of the matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ 1 & 5 & 0 \\ 2 & 5 & 1 \end{bmatrix}$.
4. Find the polar equation of the Cartesian coordinate $x^2 + y^2 = 1$.
5. Graph the set of points whose polar coordinates satisfy the condition $-3 \leq r \leq 2,\ \theta = \frac{\pi}{4}$?

(4×1=4)

PART – B
Short Essay

Answer **any seven** questions out of ten questions. **Each** question carries **2** marks :

6. Find the derivative of $y = \frac{\sin t}{\sin 2t}$.
7. If $x = \sec t$ and $y = \tan t$, then find $\frac{dy}{dx}$.
8. Find $\frac{dy}{dx}$ when $x^3 + y^3 = 3axy$.

P.T.O.

9. Verify Mean value theorem for $f(x) = x^2$ in $[-2, 3]$.

10. Find $\underset{x \to 0}{\text{Lt}} \left[\dfrac{x - \sin x}{x^3}\right]$.

11. For what values of λ the matrix $A = \begin{bmatrix} 2 & 0 & 0 & 1 \\ 0 & 5 & 0 & 0 \\ 1 & 2 & 1 & \lambda \end{bmatrix}$ has rank 3 ? Give reason for your answer.

12. Using the Gauss-Jordan method, find the inverse of $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$.

13. Verify that the matrix $A = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$ is orthogonal.

14. Find $\dfrac{ds}{dy}$ for the curve $ay^2 = x$, where s is the arc length.

15. Find a spherical coordinate equation for the sphere $x^2 + y^2 + (z - 2)^2 = 4$. **(7×2=14)**

PART – C
Essay

Answer **any four** questions out of seven questions. **Each** question carries **3** marks :

16. Find $\dfrac{d}{d\theta}(\sin^{-1}\theta)$, where $\theta \in (-1, 1)$.

17. If $y = [x^{\tan x} + (\sin x)^{\cos x}]$, then find $\dfrac{dy}{dx}$.

18. Find the values of a and b such that $\underset{x \to 0}{\text{Lt}} \left[\dfrac{x(1 + a\cos x) - b\sin x}{x^3}\right] = 1$.

19. Verify Rolle's theorem for $f(x) = \dfrac{\sin x}{e^x}$ in $[0, \pi]$.

20. Solve the equations $3x + y + 2z = 3$, $2x - 3y - z = -3$, $x + 2y + z = 4$ by matrix method.

21. Are the vectors $x_1 = (1, 1, 1)$, $x_2 = (2, 2, 2)$ and $x_3 = (3, 3, 3)$ linearly dependent ? If so express one of these as a linear combination of the others.

22. Prove that the radius of curvature at the point $\left(\dfrac{3a}{2}, \dfrac{3a}{2}\right)$ of the Folium $x^3 + y^3 = 3axy$ is $\dfrac{3a}{8\sqrt{2}}$.

(4×3=12)

PART – D
Long Essay

Answer **any two** questions out of four questions. **Each** question carries **5** marks :

23. a) If $\sin y = x \sin(a + y)$, then find $\frac{dy}{dx}$.

    b) If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

24. a) Expand $\log(1 + x)$ upto the term containing x^5.

    b) Expand $\log(1 + \sin^2 x)$ in powers of x as far as the term in x^6.

25. a) Reduce the matrix $A = \begin{bmatrix} 4 & 8 & 8 & 0 \\ 1 & 3 & 4 & 0 \\ 2 & 2 & 4 & 2 \end{bmatrix}$ into its normal form and hence find its rank.

    b) Test for consistency of the linear system of equations $x + 2y + 4z + w = 5$, $3x + 6y + 12z + 3w = 15$, $4x + 8y + 16z + 4w = 0$, $5x + 10y + 20z + 5w = 0$.

26. a) Show that the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ is $4a \cos\frac{\theta}{2}$.

    b) Find all polar coordinates of the point $\left(2, \frac{\pi}{6}\right)$. **(2×5=10)**