

**K17U 1119**

Reg. No. : .....

Name : .....

**II Semester B.Sc. Degree (C.C.S.S. – Supple.) Examination, May 2017**  
**COMPLEMENTARY COURSE IN MATHEMATICS**  
**2C02 MAT : Differential and Integral Calculus**  
**(2009-13 Admns.)**

Time : 3 Hours

Max. Weightage : 30

1. Fill in the blanks :

- a)  $\frac{d}{dx}(\operatorname{sech} x) =$  \_\_\_\_\_
- b) The differential coefficient of  $\log \tan x$  is \_\_\_\_\_
- c) The differential coefficient of  $f(\log x)$  where  $f(x) = \log x$  is \_\_\_\_\_
- d) If  $x = a(1 - \cos \theta)$ ,  $y = a(\theta - \sin \theta)$ , then  $\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}}$  is \_\_\_\_\_ (Weightage 1)

Answer any six from the following (weightage 1 each) :

2. Determine the limit of  $\frac{\log(x-a)}{\log(e^x - e^0)}$  as  $x \rightarrow a$ .
3. Find the volume of the solid obtained by revolving one arc of the cycloid  $x = a(\theta + \sin \theta)$ ,  $y = a(1 + \cos \theta)$ .
4. State Maclaurin's theorem.
5. The function  $f(x)$  is defined in  $[0, 1]$  as follows  $f(x) = 1$  for  $0 < x < \frac{1}{2}$   
 $= 2$  for  $\frac{1}{2} \leq x \leq 1$

Show that  $f(x)$  satisfies none of the conditions of Rolle's theorem, yet  $f'(x) = 0$  for many points in  $[0, 1]$ .

P.T.O.

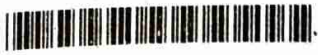


6. Find the radius of curvature of a curve  $x = a(t - \sin t)$ ,  $y = a(1 - \cos t)$  using equation in parametric coordinates.
7. State and prove the Euler's theorem on homogeneous functions.
8. Find the area of the cardioid  $r = a(1 - \cos \theta)$ .
9. Find the whole length of the asteroid  $x^{2/3} + y^{2/3} = a^{2/3}$ .
10. 
$$\int_1^{\sqrt{2}} \int_{-\sqrt{4-2y^2}}^{\sqrt{4-2y^2}} y dx dy.$$
 (Weightage : 6×1=6)

Answer **any seven** from the following (weightage **2 each**).

11. Change the order of integration in  $\int_0^a \int_y^a \frac{x}{x^2 + y^2} dx dy$  and hence evaluate the same.
12. The area included between the curve  $y^2 = x^3$  and  $x^2 = y^3$  is rotated about the x-axis. Find the volume of the solid generated.
13. Expand  $\sin x$  in powers of  $\left(x - \frac{\pi}{2}\right)$ .
14. If  $u = \log (\tan x + \tan y + \tan z)$ . Show that  $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$
15. Find the surface of the solid formed by revolving the cardioid  $r = a(1 + \cos \theta)$  about the initial line.
16. Find the reduction formula for  $\int \sin^n x dx$ .
17. If  $f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2}$ ,  $f'(0) = a$  and  $f(0) = b$ , find  $f''(x)$  where  $y$  is independent of  $x$ .
18. Find by double integration, the area lying between the parabola  $y = 4x - x^2$  and the line  $y = x$ .





19. If  $y = e^{\tan^{-1}x}$ , prove that  $(1 + x^2)y_{n+2} + (2nx + 2x - 1)y_{n+1} + n(n + 1)y_n = 0$ .

20. Find  $\frac{dz}{dt}$  when  $z = xy^2 + x^2y$ ,  $x = at^2$ ,  $y = 2at$ ,

(Weightage :  $2 \times 7 = 14$ )

Answer **any three** from the following (weightage **3 each**).

21. Find the volume bounded by the coordinate planes and the plane  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ .

22. Show if  $y = \sin(m \sin^{-1}x)$ , then  $(1 - x^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} + m^2y = 0$ . Hence expand  $\sin m\theta$  in powers of  $\sin \theta$ .

23. Show that the evolute of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is  $(ax)^{\frac{2}{3}} + (by)^{\frac{2}{3}} = (a^2 - b^2)^{\frac{2}{3}}$ .

24. If  $u = \log(x^3 + y^3 + z^3 - 3xyz)$ , show that  $\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = \frac{-9}{(x + y + z)^2}$ .

25. Find by double integration the area of the region enclosed by the curves  $x^2 + y^2 = a^2$ ,  $x + y = a$  in the first quadrant. (Weightage :  $3 \times 3 = 9$ )

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