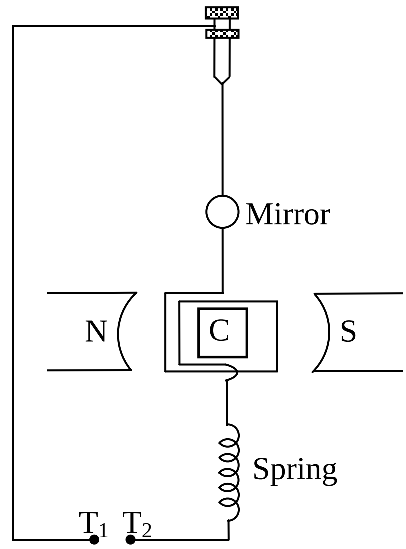
Theory of moving coil ballistic galvanometer

Moving coil ballistic galvanometer consists of a rectangular coil of thin copper wire wound on a non-metallic frame of ivory. It is suspended by means of a phosphor bronze wire between the pole pieces of a powerful horse-shoe magnet. A small circular mirror is attached to the suspension wire. The lower end of the rectangular coil is connected to a hair-spring. The upper end of the suspension wire and the lower end of the spring are connected to the terminals T_1 and T_2. C is a cylindrical soft iron core placed inside the coil symmetrically with the grooves of the pole pieces. This iron core concentrates the magnetic field and helps in producing the radial field.



Principle: A rectangular coil suspended in a magnetic field experiences a torque when a current flows through it is the basic principle of a moving coil galvanometer.

The BG is used to measure electric charge so that the current is always momentary. This produces only an impulse on the coil and a throw is registered. The oscillations of the coil are practically undamped and the period of oscillation is fairly large.

Theory: Let N be the number of turns of the rectangular coil which encloses an area A. The coil is suspended in a magnetic induction B. The torque acting on the coil when a current i is flowing through it is given by,

$$\text{Torque on the coil} \quad \tau = NiBA = C\theta \qquad (1)$$

If the current flows for a short interval of time dt, the angular impulse produced in the coil is,

$$\tau dt = NBAidt = NBAdq$$

If the current passes for t seconds, the total angular impulse is given by,

$$\int_0^t \tau dt = NBA\int_0^t dq = NBAq \qquad (2)$$

Let I be the moment of inertia of the coil about the axis of suspension and ω is the angular velocity attained by the coil. The change in angular momentum is due to the angular impulse. Thus,

$$I\omega = NBAq \qquad (3)$$

The kinetic energy $\frac{1}{2}I\omega^2$ of the system is completely used to twist the suspension wire through an angle θ. Let C be the restoring couple per unit twist of the suspension wire. Then the work done in twisting the wire through an angle θ is given by $\frac{1}{2}C\theta^2$. Then,

$$\frac{1}{2}I\omega^2 = \frac{1}{2}C\theta^2$$

$$\therefore \quad I\omega^2 = C\theta^2 \qquad (4)$$

The period of oscillation of the coil is given by,

$$T = 2\pi\sqrt{\frac{I}{C}}$$

i.e
$$I = \frac{T^2C}{4\pi^2} \tag{5}$$

Multiplying eqns. 4 and 5 we get,

$$I^2\omega^2 = \frac{T^2C^2\theta^2}{4\pi^2}$$

i.e.
$$I\omega = \frac{TC\theta}{2\pi} \tag{6}$$

Using eqn.6 in eqn.3 we get,

$$\frac{TC\theta}{2\pi} = NABq$$

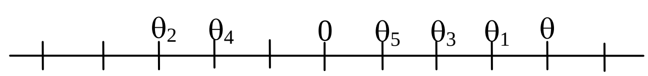
$$\therefore \qquad q = \left(\frac{T}{2\pi}\right)\left(\frac{C}{NAB}\right)\theta \tag{7}$$

Eqn.7 gives the relationship between the charge flowing through the coil and the ballistic throw of the galvanometer. Eqn.7 can be written as,

$$q = K\theta \tag{8}$$

where, $K=\left(\frac{T}{2\pi}\right)\left(\frac{C}{NAB}\right)$ is called the *ballistic reduction factor*.

Damping correction: While deriving eqn.7 we have assumed that the whole kinetic energy imparted to the coil is completely used to twist the suspension wire. This is not exactly true. In actual practice the motion of the coil is damped by air resistance and induced current in the coil. The first throw of the galvanometer is, therefore, smaller than it would have been in the absence of damping. So it is necessary to apply a correction for damping to the first throw of the galvanometer.



Let θ_1, θ_2, θ_3, …… be the successive maximum deflections from the zero position to the right and left as shown in the figure. Then it is found that,

$$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4}\ldots\ldots = d \tag{9}$$

where, d is a constant called the *decrement* per half vibration. Let $d = e^{\lambda}$. Then, $\lambda = \log_e d$. Here λ is called as the *logarithmic decrement*. Thus for a complete oscillation,

$$\frac{\theta_1}{\theta_3} = \frac{\theta_1}{\theta_2}\times\frac{\theta_2}{\theta_3} = d^2 = e^{2\lambda} \tag{10}$$

$\therefore$ For one fourth of an oscillation the RHS of eqn.10 is $e^{\lambda/2}$. Let θ be the true first throw in the absence of damping. The actual first throw in presence of damping is θ_1, which is observed after the coil completes a quarter of vibration. Thus the decrement for θ is given by,

$$\frac{\theta}{\theta_1} = e^{\lambda/2} \tag{10a}$$

Taking fourth root of eqn.10, we get,

$$\left(\frac{\theta_1}{\theta_3}\right)^{1/4} = e^{\lambda/2} = \frac{\theta}{\theta_1} \text{ ; eqn.10a is used here.}$$

$$\therefore \qquad \theta = \theta_1\left(\frac{\theta_1}{\theta_3}\right)^{1/4} \tag{11a}$$

Or approximately,

$$\frac{\theta}{\theta_1} = e^{\lambda/2} \approx \left(1+\frac{\lambda}{2}\right)$$

$$\therefore \qquad \theta = \theta_1\left(1+\frac{\lambda}{2}\right) \tag{11b}$$

We calculate λ by observing the first 11 consecutive throws. Then,

$$\frac{\theta_1}{\theta_{11}} = \frac{\theta_1}{\theta_2}\times\frac{\theta_2}{\theta_3}\times\frac{\theta_3}{\theta_4}\times\frac{\theta_4}{\theta_5}\times\ldots\ldots\ \ldots\times\frac{\theta_9}{\theta_{10}}\times\frac{\theta_{10}}{\theta_{11}} = e^{10\lambda}$$

$$\text{i.e.} \qquad 10\lambda = \log_e\left(\frac{\theta_1}{\theta_{11}}\right)$$

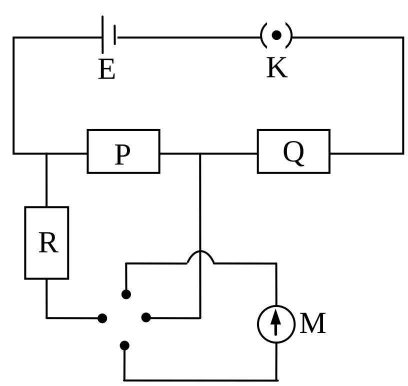
$$\therefore \qquad \lambda = \frac{1}{10}\log_e\left(\frac{\theta_1}{\theta_{11}}\right) = \frac{2.3026}{10}\log_{10}\left(\frac{\theta_1}{\theta_{11}}\right) \tag{12}$$

Then eqn.7 corrected for damping is given by,

$$q = \left(\frac{T}{2\pi}\right)\left(\frac{C}{NAB}\right)\theta_1\left(1+\frac{\lambda}{2}\right) \tag{13}$$

Current, charge and voltage sensitivities of a moving coil galvanometer

Current sensitivity (Figure of merit): The figure of merit or current sensitivity of a moving coil galvanometer is the current required to produce a deflection of 1 mm on a scale kept at a distance of 1 m from the mirror. It is expressed in μA/mm.



From eqn.1, current sensitivity $= \dfrac{i}{\theta} = \dfrac{C}{NAB}$ (14)

Charge sensitivity: The charge sensitivity (the *ballistic reduction factor*) of a moving coil galvanometer is the charge (transient current) required to produce a deflection (throw or kick) of 1 mm on a scale kept at a distance of 1 m from the mirror. By eqn.7 and 8, Charge sensitivity, $K = \frac{q}{\theta} = \left(\frac{T}{2\pi}\right)\left(\frac{C}{NAB}\right) = \frac{T}{2\pi}\times\text{current sensitivity}$ (15)

Some authors defined the current sensitivity as the deflection produced by unit current on a scale kept at a distance 1m from the mirror of the galvanometer.

$$\text{Current sensitivity} = \frac{\theta}{i}$$

With this definition the figure of merit is the reciprocal of the current sensitivity.

Voltage sensitivity: The voltage sensitivity is the potential difference that should be applied to the galvanometer to produce a deflection of 1 mm on a scale placed at a distance of 1 m from the mirror. It is expressed in μV/mm.

Figure shows the electrical circuit for the determination of current and voltage sensitivities of a moving coil galvanometer. Resistance boxes P and Q connected in series with the lead accumulator E form a potential divider arrangement. Potential difference developed across P is applied to the moving coil galvanometer through the resistance R and the commutator. A low resistance, say 1 Ω, is introduced in P and a high resistance, say 9999 Ω, in Q. The deflection produced θ is determined. Then,

$$\text{Voltage sensitivity, } S_v = \frac{EP}{(P+Q)\theta}\times 10^6\ \mu\text{V/mm} \quad (16)$$

Now resistance in R is adjusted such that the deflection becomes $\theta/2$. The resistance R is, then, equal to the galvanometer resistance R_g.

$$\text{Current sensitivity, } S_c = \frac{EP}{(P+Q)R_g\theta}\times 10^6\ \mu\text{A/mm} \quad (17)$$

The experiment is repeated for different values of P keeping P+Q equal to 10000 Ω.

(Remember, θ is originally defined as the angle of deflection of the mirror. When the mirror turns through an angle θ, the reflected ray turns through an angle 2θ. Since the distance between the mirror and the scale is 1 m, angle in radian = arc length in metre. Thus, if θ is taken as the scale reading in millimeter, multiplication with 2 is needed in the calculations of charge sensitivity, current sensitivity and voltage sensitivity).

Let d be the deflection in mm on the scale, then, $\phi = 2\theta = \frac{d\times 10^{-3}}{D}$ radian $= d\times 10^{-3}$ radian

Since D = 1 m

Then, $\frac{I}{\phi} = \frac{I\times 10^3}{d}$ ampere per metre $= \frac{I}{d}$ ampere per millimeter. Thus in our further discussion we may use ϕ or θ in place of d in mm, as the deflection of the spot of light.

Electromagnetic damping in a moving coil galvanometer

A moving coil galvanometer consists of a rectangular current carrying coil suspended in a uniform radial magnetic field. The current through the coil produces a torque which tries to rotate the coil. A restoring torque, provided by the stiffness of the suspension, is set up in the system and it counterbalances the electromagnetic torque.

Damping factors

1. Viscous drag of air and mechanical friction. The damping couple due to this is proportional to the angular velocity of the system. Usually it is very small and is neglected.
2. Induced currents in the neighboring conductors. These currents produce two types of damping couple, the open circuit damping couple and the closed circuit damping couple. The former one, according to the law of electromagnetic induction, is proportional to the angular velocity and is represented by $-b\frac{d\theta}{dt}$, where, b is the damping coefficient. The latter one is directly proportional to the angular velocity and inversely proportional to the resistance of the circuit. It is given by, $-\frac{\xi}{R}\frac{d\theta}{dt}$, where, ξ involves all the coil constants such as area, magnetic flux and so on.

If C is the restoring couple per unit twist of the suspension and I is the moment of inertia of the oscillating system, the equation of motion is given by,

$$I\frac{d^2\theta}{dt^2} = -C\theta - b\frac{d\theta}{dt} - \frac{\xi}{R}\frac{d\theta}{dt}$$

$$\frac{d^2\theta}{dt^2} + \frac{1}{I}\left(b+\frac{\xi}{R}\right)\frac{d\theta}{dt} + \frac{C}{I}\theta = 0$$

Now put $\frac{1}{I}\left(b+\frac{\xi}{R}\right) = \gamma_e$ and $\frac{C}{I} = \omega_0^2$ (18)

Then, $$\frac{d^2\theta}{dt^2} + \gamma_e\frac{d\theta}{dt} + \omega_0^2\theta = 0 \quad (19)$$

This equation is similar to the equation for a damped harmonic oscillator given by,

$$\ddot{x} + 2r\dot{x} + \omega_0^2 x = 0$$

Comparing these equations we get, $x = \theta$, $2r = \gamma_e$ and $\omega_0^2 = \frac{C}{I}$

For damped oscillations, we have, $x = e^{-rt}\left\{C_1 e^{\left(\sqrt{r^2-\omega_0^2}\right)t} + C_2 e^{\left(-\sqrt{r^2-\omega_0^2}\right)t}\right\}$

Thus, $$\theta = e^{-\frac{\gamma_e}{2}t}\left\{C_1 e^{\left(\sqrt{\frac{\gamma_e^2}{4}-\omega_0^2}\right)t} + C_2 e^{\left(-\sqrt{\frac{\gamma_e^2}{4}-\omega_0^2}\right)t}\right\} \quad (20)$$

where, C_1 and C_2 are undetermined constants to be evaluated from the initial conditions.

Case 1: Non-oscillatory, aperiodic or dead beat motion

If the damping is high such that $\frac{\gamma_e^2}{4} > \omega_0^2$ the exponents of eqn.20 are real and the motion of the system is non-oscillatory. The angular displacement decays according to eqn.20. This is the case of dead beat motion. The requirements of a dead beat galvanometer are,

1. The moment of inertia I of the system must be small.
2. The electromagnetic rotational resistance 'b' should be large and the suspension is not fine.
3. The term ξ, which involves all coil constants, should be large and the coil should be wound on a conducting frame.
4. The resistance R must be small.

Case 2: Critical damping

When $\frac{\gamma_e^2}{4} = \omega_0^2$, the galvanometer is said to be critically damped. The motion of the coil is non-oscillatory and it comes to rest in a minimum time after deflection.

Case 3: Light damping: Ballistic motion

When $\frac{\gamma_e^2}{4} < \omega_0^2$, the exponents of the bracketed terms of eqn.20 are imaginary. Hence the motion of the coil is oscillatory in this case. The solution is given by,

$$\theta = e^{-\frac{\gamma_e}{2}t}\left\{C_1 e^{\left(i\sqrt{\omega_0^2-\frac{\gamma_e^2}{4}}\right)t} + C_2 e^{\left(-i\sqrt{\omega_0^2-\frac{\gamma_e^2}{4}}\right)t}\right\}$$

Or, $$\theta = Q_0\, e^{-\frac{\gamma_e}{2}t} \sin\left\{\left(\sqrt{\frac{C}{I}-\frac{\gamma_e^2}{4}}\right)t + \phi_0\right\}$$

$$= Q_0\, e^{-\frac{\gamma_e}{2}t} \sin\left(qt + \phi_0\right)$$

where, $$q = \sqrt{\omega_0^2 - \frac{\gamma_e^2}{4}} = \sqrt{\frac{C}{I} - \frac{1}{4I^2}\left(b+\frac{\xi}{R}\right)^2}$$

The motion of the galvanometer is said to be ballistic when the damping factor $\frac{\gamma_e}{2} = \frac{1}{2I}\left(b+\frac{\xi}{R}\right)$ is small. The requirements of a ballistic galvanometer are reverse of those for a dead beat galvanometer and are,

1. The moment of inertia I of the system must be large.
2. The electromagnetic rotational resistance 'b' is small and the suspension is fine.
3. The term ξ, which involves all coil constants, should be small and the coil should be wound on a non-conducting frame like wood or paper.
4. The resistance R must be large.

Uses of ballistic galvanometers

1. To compare capacities of capacitors.
2. To compare e m f of cells.
3. To find self and mutual inductance of coils.
4. To find the magnetic flux or to find the intensity of a magnetic field.
5. To find the angle of dip at a place using earth inductor.
6. To find a high resistance, by method of leakage through a capacitor.

Exp.No.2.18

Mirror Galvanometer-Figure of Merit

Aim: To determine the figure of merit of a moving coil mirror galvanometer.

Apparatus: Power supply, three resistance boxes, commutator key, ordinary keys, lamp and scale arrangement.

Theory: The figure of merit of a moving coil galvanometer is the current required to produce a deflection of 1 mm on a scale kept at a distance of 1 m from the mirror. It is expressed in μA/mm. (Some authors called it as current sensitivity). If a current I ampere produces a deflection 'θ' of the spot of light on the scale kept at a distance 1 m from mirror, the figure of merit of the moving coil galvanometer is,

$$k = \frac{I}{\theta} \qquad (1)$$

The current I through the galvanometer is calculated as follows. Since R + G >> P, the current through P and Q is

$$I' = \frac{E}{P+Q} \qquad (2)$$

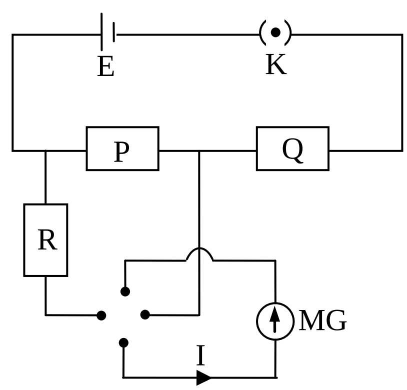


Fig.a

$$\text{Voltage across P} = V' = I'P = \frac{EP}{P+Q}$$

$$\therefore \quad I = \frac{V'}{R+G} = \frac{EP}{(P+Q)(R+G)} \qquad (3)$$

When R = 0,

$$I = \frac{EP}{(P+Q)G} \qquad (4)$$

$$\therefore \ \textit{Figure of merit},\ k = \frac{I}{\theta} = \frac{E}{(P+Q)G}\left(\frac{P}{\theta}\right) \qquad (5)$$

When a resistance is introduced in R, the current I decreases and hence the deflection in the galvanometer also decreases. The current and hence the galvanometer deflection reduces to half when

$$R \approx G + P \approx G \qquad (6)$$

(See the appendix of exp.no.2.15).

Procedure: The open circuit voltage of the power supply E is adjusted to 2 volt and is measured. Let it be E. It is then connected to the circuit as shown in the fig.b. The galvanometer is connected across P through a resistance box R in series with it. Introduce suitable resistances in P and Q and no resistance in R. The lamp and scale arrangement is adjusted such that the distance between the mirror and the scale is 1 m and the spot of light is obtained on the zero line.

To begin with the experiment a small resistance, say 1 ohm, is introduced in P and a resistance 9999 ohm in Q, so that P + Q = 10,000 Ω. Ensure that all the unplugged keys are tightly locked. Then the damping key K_2 is made open and the voltage across P is applied to the galvanometer. The steady current through the galvanometer produces a steady deflection of the spot of light to one side. The deflection θ_1 is noted. Now introduce suitable resistance in R and find out the resistance in R required to reduce the deflection to half of the initial value ($\theta_1/2$). Using commutator key the current through the galvanometer is reversed. Again, the deflection and the resistance in R for half deflection are noted. The value of R for half deflection gives the galvanometer resistance G.

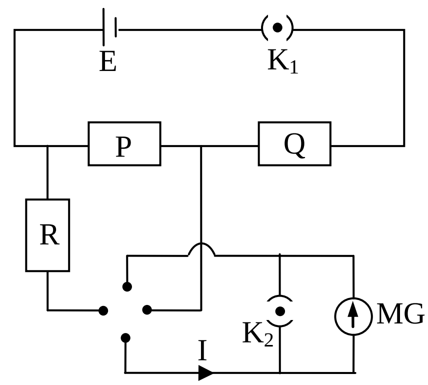


Fig.b

The experiment is repeated for P = 2, 3, 4, ……. Keeping P + Q = 10,000 Ω (for our convenience of calculation only). In each case the deflection θ and the resistance R needed for half deflection are found out. Figure of merit is calculated using eqn.5.

Precautions

- Measure the open circuit voltage E of the power supply. Only a 2 volt potential difference is required for this experiment.
- Ensure that the distance between the mirror and the scale is 1 m, since the definition of figure of merit demands that.
- Ensure that the voltage applied to the galvanometer is the voltage across the much smaller resistance P.
- Ensure that a damping key is connected to protect the galvanometer. The damping key is connected such that when it is closed, the galvanometer becomes short circuited and the deflection is reduced to zero.
- The circuit is closed only after taking the required resistances in the boxes.
- Ensure P << Q. Also P << R +G. Refer exp. No. 2.15 and its appendix.

Observation and tabulation

E m f (open circuit voltage) of the power supply, E = ………… volt

P ohm	Q ohm	Deflection θ in mm to			Resistance G for half deflection			$k = \frac{E}{(P+Q)G}\left(\frac{P}{\theta}\right)$ ampere/mm
		Left mm	Right mm	Mean mm	Left ohm	Right ohm	Mean ohm	
							Mean		

Result

Resistance of the moving coil mirror galvanometer, G = …….. ohm

Figure of merit of the galvanometer k = …….. ampere/mm

Exp.No.2.19

Ballistic Galvanometer- Ballistic constant- using Hibbert's Magnetic Standard (H M S)

Aim: To determine the ballistic constant of a ballistic galvanometer using Hibbert's magnetic standard.

Apparatus: The ballistic galvanometer, H M S, power supply, resistance box and a commutator.

Theory: A **ballistic galvanometer** is a type of mirror galvanometer. Unlike a current-measuring galvanometer, the moving part has a large moment of inertia, thus giving it a long oscillation period. It can be used to measure the quantity of charge discharged through it. It can be either of the moving coil or moving magnet type. Its deflection (throw of spot of light) is proportional to the charge given to it.

$$Q \propto \theta$$

$$Q = K\theta \qquad (1)$$

where, K is a constant, called the Ballistic Constant, for the given ballistic galvanometer. (Refer eqns.7, 8 and 15 of the theory of moving coil galvanometer).

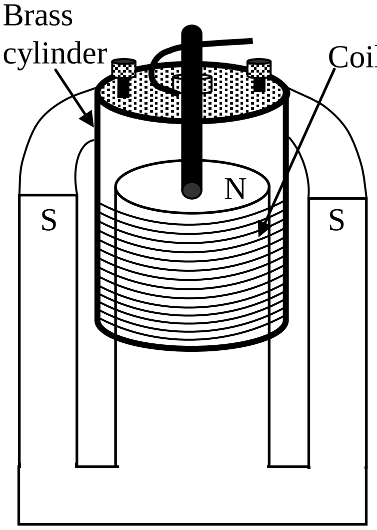


Fig.a: H M S

The Hibbert's magnetic standard consists of a coil of 'n' turns of fine insulated copper wire wound over a hollow brass cylinder. It can move freely through the groove of a permanent magnet made of steel. The north pole of the magnet is cylindrical in shape. It is surrounded by the south Pole having hollow cylindrical shape. The coil can be raised and held in a position just above the pole pieces with a lever arrangement. When the lever is pressed, the coil falls down through the magnetic field. Thus the magnetic flux linked with the coil increases from 0 to a maximum of ϕ, which is marked on the instrument.

Let ϕ' be the flux linked with the single turn of the coil at any instant of time. Therefore the flux linked with the coil is $n\phi'$. Then by Faraday's law of induction (flux rule),

Induced e m f, $\varepsilon = -n\dfrac{d\phi'}{dt}$ (2)

Induced current, $I = \dfrac{\varepsilon}{R+G}$

$$= \left(\frac{n}{R+G}\right)\frac{d\phi'}{dt}$$

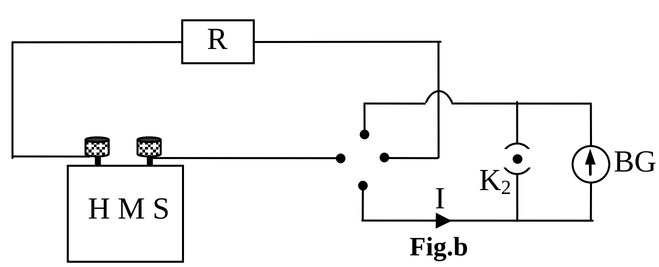


Fig.b

where, G is the galvanometer resistance. Negative sign is avoided since it indicates only the direction of the current.

Total induced charge, $$Q = \int I dt = \left(\frac{n}{R+G}\right)\int_0^{\phi} d\phi' = \frac{n\phi}{R+G} \qquad (3)$$

Let θ be the corrected throw of the spot of light. Then by eqn.1,

$$\frac{n\phi}{R+G} = K\theta$$

$$\frac{1}{\theta} = \frac{K(R+G)}{n\phi} \tag{4}$$

If θ' is the throw when the resistance R is changed to R',

$$\frac{n\phi}{R'+G} = K\theta'$$

$$\frac{1}{\theta'} = \frac{K(R'+G)}{n\phi} \tag{5}$$

Subtracting eqn.5 from eqn.4,

$$\frac{1}{\theta}-\frac{1}{\theta'} = \frac{K(R-R')}{n\phi}$$

$$\therefore \qquad K = \frac{n\phi}{(R-R')}\left(\frac{1}{\theta}-\frac{1}{\theta'}\right) \tag{6}$$

Procedure: The Hibbert's magnetic standard is connected to the Ballistic galvanometer through a resistance box R and a commutator as shown in fig.b. A suitable resistance R is introduced in the resistance box. The brass cylinder containing the coil is raised and is kept in position by the lever arrangement. The lamp and scale is arranged such that the distance between the mirror and scale is 1 m and the spot of light is stationary at zero line. (The spot of light can be made stationary by closing the damping key). Now the damping key is made open. Then the lever arrangement is released by pressing it and the brass cylinder is allowed to fall down. The first throw θ_1 and the next throw θ_3 of the spot of light on the same side are noted. The corrected throw is calculated as $\theta = \theta_1\left(\frac{\theta_1}{\theta_3}\right)^{1/4}$. (Refer eqn.11a of the theory of the ballistic galvanometer).

The experiment is repeated after reversing the commutator key and the throw in the other direction is determined. The mean throw is calculated. Now change the resistance R to R' and find out the corrected θ' by the same method as discussed above. Repeat the experiment for different values of resistance R. ϕ and n of H M S written on the apparatus are noted. (Usually ϕ is written in c g s unit. Multiply it with 10^{-8} to convert to S I system). Then by using eqn.6, the ballistic constant is calculated for different sets of resistances as R and R'.

Precautions

- Initially sufficiently large resistance must be taken so as to ensure that the deflection is within the scale.
- Due to the shorting of BG through the coil, it will be damped. Hence there will be no oscillations of the spot light. This can be avoided by making the commutator key open immediately after the first kick begins.
- If the throw is expressed in millimeter, the K has unit coulomb/mm.
- There may be some variation in the kick with our speed of action.

Observation and tabulation

Determination of throw with different resistances

Sl.No.	Resistance in box Ohm	Mean throw in the galvanometer						Mean corrected throw mm
		Left			Right			
		θ_1 mm	θ_3 mm	$\theta = \theta_1 \left(\frac{\theta_1}{\theta_3}\right)^{1/4}$ mm	θ_1 mm	θ_3 mm	$\theta = \theta_1 \left(\frac{\theta_1}{\theta_3}\right)^{1/4}$ mm	
1								
2								
3								
4								
5								
6								

Magnetic flux of H M S, ϕ = ………. c g s unit = ……. $\times 10^{-8}$ weber

Number of turns of the coil of H M S, n = ………

Calculation of Ballistic constant

Sl.No.	R ohm	R' ohm	θ mm	θ' mm	$K = \frac{n\phi}{(R - R')}\left(\frac{1}{\theta} - \frac{1}{\theta'}\right)$ coulomb/mm
1					
2					
3					
4					
				Mean K	

Result

The ballistic constant of the given ballistic galvanometer, K= ……….. coulomb/mm

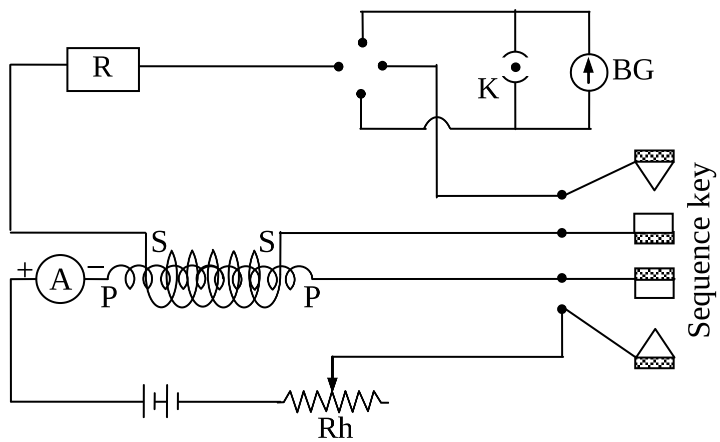
Exp.No.2.20

Ballistic Galvanometer- ballistic constant using standard solenoid

Aim: To determine the ballistic constant (ballistic reduction factor or charge sensitivity) of a ballistic galvanometer using a standard solenoid inductor.

Apparatus: The ballistic galvanometer, The standard solenoid inductor, power supply, resistance box, rheostat, ammeter, a commutator key (reversing key), a sequence key etc.

Theory: The deflection (throw of spot of light) in the ballistic galvanometer is proportional to the charge given to it.



$$Q \propto \theta$$

$$Q = K\theta \qquad (1)$$

where, K is a constant, called the Ballistic Constant, for the given ballistic galvanometer. (Refer eqns.7, 8 and 15 of the theory of moving coil galvanometer).

When the sequence key is pressed, the current through the primary increases from 0 to I. Hence the magnetic flux associated with it also increases from 0 to ϕ_P. Let I' be the instantaneous current through the primary coil. Since the primary coil is a long solenoid, there is magnetic field only inside the coil and is given by,

$$B = \mu_0 n I' \qquad (2)$$

where, n is the number of turns per unit length of the primary coil.

The secondary coil is placed in this magnetic field. The instantaneous magnetic flux linked with single turn of the secondary coil is (assuming there is tight coupling and hence the coefficient of coupling is one),

$$\phi' = BA \qquad (3)$$

where, A is the area of the secondary through which the magnetic field passes. This area A is equal to the area of the secondary if the secondary is well inside the primary and it is equal to the area of the primary if the secondary winding is outside the primary winding.

Let N be the total number of turns of the secondary. Therefore, the flux linked with the secondary coil is,

$$N\phi' = NAB = \mu_0 NnAI' \qquad (4)$$

Then by Faraday's law of induction (flux rule),

$$\text{Induced e m f in the secondary,} \quad \varepsilon = -N\frac{d\phi'}{dt} \qquad (5)$$

$$\text{Induced current in the secondary,} \quad I_s = \frac{\varepsilon}{R+G+S} = \left(\frac{N}{R+G+S}\right)\frac{d\phi'}{dt}$$

Using eqn.4, $$= \left(\frac{\mu_0 NnA}{R+G+S}\right)\frac{dI'}{dt} \qquad (6)$$

where, G is the galvanometer resistance and S is the resistance of the secondary. Negative sign is avoided since it indicates only the direction of the current.

Total induced charge in the secondary, $$Q = \int I_s dt = \left(\frac{\mu_0 NnA}{R+G+S}\right)\int_0^I dI'$$

$$= \frac{\mu_0 NnAI}{R+G+S} \qquad (7)$$

Let θ be the corrected throw of the spot of light. Then by eqn.1,

$$\frac{\mu_0 NnAI}{R+G+S} = K\theta$$

$$\frac{1}{\theta} = \frac{K(R+G+S)}{\mu_0 NnAI} \qquad (8)$$

If θ′ is the throw when the resistance R is changed to R′,

$$\frac{1}{\theta'} = \frac{K(R'+G+S)}{\mu_0 NnAI} \qquad (9)$$

Subtracting eqn.9 from eqn.8,

$$\frac{1}{\theta} - \frac{1}{\theta'} = \frac{K(R-R')}{\mu_0 NnAI}$$

$$\therefore \qquad K = \frac{\mu_0 NnAI}{(R-R')\left(\frac{1}{\theta}-\frac{1}{\theta'}\right)} \qquad (10)$$

We get the same result if the current in the primary is collapsed from I to 0.

Procedure: The connections are made as shown in the figure. The standard solenoid is a long solenoid of large number of turns. The secondary has lesser number of turns and is much smaller than the primary. It is arranged coaxially at the middle of the primary. Its diameter may be greater or lesser than the primary.

The primary coil, an ammeter and the rheostat are connected in series with the power supply through the sequence key. The secondary coil, resistance box are connected to the BG through the commutator key and the sequence key. The lamp and scale is arranged such that the distance between the mirror and scale is 1 m and the spot of light is stationary at zero line. (The spot of light can be made stationary by closing the damping key).

To begin with the experiment, introduce a sufficiently large resistance R in the box. The sequence key is kept pressed and the rheostat is adjusted for a current of 0.5 ampere. Now the damping key is made open. The sequence key is then released. The first throw θ_1 and the next throw θ_3 of the spot of light on the same side are noted. The corrected throw is calculated as $\theta = \theta_1\left(\frac{\theta_1}{\theta_3}\right)^{1/4}$. (Refer eqn.11a of the theory of the ballistic galvanometer).

The experiment is repeated after reversing the commutator key and the throw in the other direction is determined. The mean throw is calculated. Now change the resistance R to R′ and find out the corrected θ' by the same method as discussed above. Repeat the experiment for different values of resistance R. The entire experiment can also be repeated for different values of current.

The number of turns per unit length 'n' of the primary coil and the total number of turns 'N' of the secondary coil are noted. The area A of the secondary through which the magnetic field passes is determined by measuring the circumference of secondary or primary as the case may be. Assuming μ_0, the ballistic constant of the galvanometer is calculated using the eqn.10.

- Care must be given to the measurement of area A. It is equal to the area of the secondary if the secondary is well inside the primary and it is equal to the area of the primary if the secondary winding is outside the primary winding.
- If the sequence key functions well there is no problem of shorting of BG since the circuit is broken when the sequence key is released. So ensure that after releasing the sequence key there is no contact between the key terminals connected to the secondary.
- Initially sufficiently large resistance must be taken so as to ensure that the deflection is within the scale.
- If the throw is expressed in millimeter, the K has unit coulomb/mm. There may be some variation in the kick with our speed of action.

Observation and tabulation

Determination of throw with different resistances for current I_1 = ……. ampere

Sl.No.	Resistance in the box Ohm	Mean throw in the galvanometer						Mean corrected throw mm
		Left			Right			
		θ_1 mm	θ_3 mm	$\theta = \theta_1\left(\frac{\theta_1}{\theta_3}\right)^{1/4}$ mm	θ_1 mm	θ_3 mm	$\theta = \theta_1\left(\frac{\theta_1}{\theta_3}\right)^{1/4}$ mm	
1								
2								
3								
4								
5								
6								

Number of turns of the primary coil, N_p = ………

Length of the primary coil, L = ……… cm = …… m

Number of turns per metre of primary, $n = \frac{N_p}{L}$ = ……… turns/metre

Total number of turns of the secondary coil, N = ………

Circumference of primary/secondary , L' = ……… cm

* (Strike off which is not applicable)

Radius of the primary/secondary, $r = \frac{L'}{2\pi}$ = …….. m

Area of the secondary through which the magnetic field passes,

$$A = \text{Area of primary/secondary} = \pi r^2 = \ldots\ldots\ \text{m}^2$$

Calculation of Ballistic constant

Sl.No.	R ohm	R′ ohm	θ mm	θ′ mm	$K = \frac{\mu_0 NnAI}{(R-R')}\left(\frac{1}{\theta}-\frac{1}{\theta'}\right)$ coulomb/mm
1					
2					
3					
4					
				Mean K	

Determination of throw with different resistances for current $I_2 =$ ……. ampere

Sl.No.	Resistance in box Ohm	Mean throw in the galvanometer						Mean corrected throw mm
		Left			Right			
		θ_1 mm	θ_3 mm	$\theta = \theta_1\left(\frac{\theta_1}{\theta_3}\right)^{1/4}$ mm	θ_1 mm	θ_3 mm	$\theta = \theta_1\left(\frac{\theta_1}{\theta_3}\right)^{1/4}$ mm	
1								
2								
3								
4								
5								
6								

Calculation of Ballistic constant

Sl.No.	R ohm	R′ ohm	θ mm	θ′ mm	$K = \frac{\mu_0 NnAI}{(R-R')}\left(\frac{1}{\theta}-\frac{1}{\theta'}\right)$ coulomb/mm
1					
2					
3					
4					
				Mean K	

Mean of K for currents I_1 and I_2 = ………… coulomb/mm

Result

The ballistic constant of the given ballistic galvanometer, K= ……….. coulomb/mm

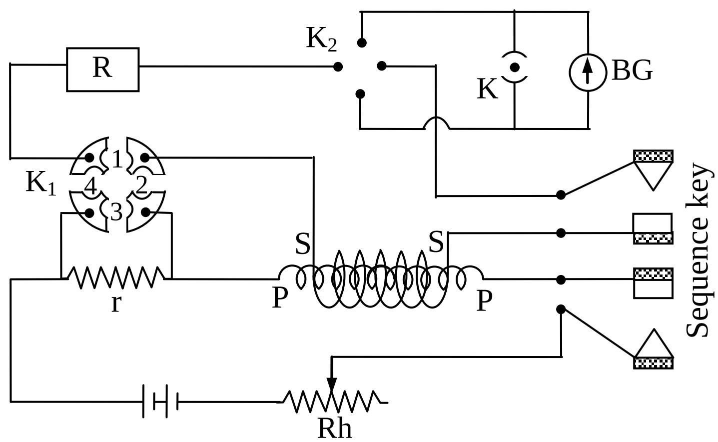
Exp.No.2.21

Ballistic Galvanometer- Determination of mutual inductance

Aim: To determine the mutual inductance between a pair of coils by Kirchoff's method.

Apparatus: The given pair of coils, the ballistic galvanometer, power supply, a rheostat, a resistance box of large resistance, a resistance box of fractional resistance or calculated length of standard copper wire (refer conversion of galvanometer into ammeter), a sequence key and two four plug commutator keys.

Theory: The mutual inductance between the two coils is defined as the change in total magnetic flux linking with one coil per unit change in current in the other coil. Or, mathematically,



$$M_{21} = N_2 \frac{d\Phi_{21}}{dI_1} = \frac{d\lambda_2}{dI_1} \qquad (1)$$

where, M_{21} is the mutual inductance of the coil-2 due to the flux produced by the coil-1 and $\lambda = N\Phi$ is the flux linkage. Again the mutual inductance is defined as

$$V_1(t) = -M_{12}\frac{dI_2}{dt} \qquad (2)$$

where, V_1 the open circuit voltage across coil-1. Similar equation can be written for the other coil. We can show that $M_{12} = M_{21} = M$. In this experiment, the e m f induced (without considering the sign) in the secondary is,

$$\varepsilon_s = M\frac{dI_p}{dt} \qquad (3)$$

Instantaneous current in the secondary,

$$I_s = \frac{\varepsilon_s}{R+G+S} = \left(\frac{M}{R+G+S}\right)\frac{dI_p}{dt} \qquad (4)$$

where, S is the resistance of the secondary coil.

Total charge flows through the BG when the current in the primary increases from 0 to I is,

$$Q = \int I_s dt = \left(\frac{M}{R+G+S}\right)\int_0^I dI_p = \frac{MI}{R+G+S} \qquad (5)$$

But, by eqn.8 of the theory of moving coil galvanometer,

$$Q = K\theta = \frac{T}{2\pi}\times k\theta \qquad \text{(Refer eqn.15 of the theory)} \qquad (6)$$

where, K is the charge sensitivity, k is the current sensitivity and θ is the corrected kick in the BG. By eqns.5 and 6,

$$\frac{MI}{R+G+S} = \frac{T}{2\pi}k\theta \qquad (7)$$

Now the voltage across the resistance ‘r’ produced by a steady current I in the primary coil is applied to the BG. Then the steady current flowing through the BG is calculated as follows.

Voltage across ‘r’ $= v = Ir$

$$\text{Current through the BG circuit} = \frac{v}{R+G+S} = \frac{Ir}{R+G+S} \tag{8}$$

This current produces a steady deflection ‘d’ in BG. Then,

$$\frac{Ir}{R+G+S} = kd \tag{9}$$

Dividing eqn.7 by eqn.9,

$$M = \left(\frac{T}{2\pi}\right)\frac{r}{d}\theta = \left(\frac{Tr}{2\pi}\right)\frac{\theta}{d} \tag{10}$$

where, the corrected throw, $\theta = \theta_1\left(\frac{\theta_1}{\theta_3}\right)^{1/4}$.

We get the same result if the current in the primary is collapsed from I to 0.

Procedure: The connections are made as shown in the figure. The primary and the secondary may be the pair of coils used in the exp.no.2.20. K_1 is a commutator key having four terminals and four gaps. The gaps are mentioned in the figure. The lamp and scale is arranged such that the distance between the mirror and scale is 1 m and the spot of light is stationary at zero line. (The spot of light can be made stationary by closing the damping key).

To begin with the experiment, insert the plug in gap 1. (Leave the other three gaps open). A suitable resistance (say, 500 Ω) is introduced in R and the rheostat also is adjusted for particular current. Press the sequence key and thus allow a steady current to flow through the primary circuit. Then unplug the damping key K. The sequence key is now released. The first throw θ_1 and the next throw θ_3 of the spot of light on the same side are noted. The corrected throw is calculated as $\theta = \theta_1\left(\frac{\theta_1}{\theta_3}\right)^{1/4}$. The experiment is repeated by reversing the commutator key K_2. The mean of the corrected throw are determined.

Next the plug in the gap 1 is released. Insert the plugs in the gaps 2 and 4. (The gaps 1 and 3 are left open). Now the voltage across ‘r’ due the same current is applied to BG. The steady deflection ‘d’ in the galvanometer is noted. The experiment is repeated after reversing key K_2 and the average of d is calculated. The entire experiment is repeated for other currents by adjusting the rheostat.

The galvanometer is made to oscillate freely. (This can be done by simply releasing one of the plugs in key K_2. Time for 20 oscillations is found for three times and the average time period T for one oscillation is determined. Calculate the mutual inductance using eqn.10.

- Initially sufficiently large resistance must be taken so as to ensure that the deflection is within the scale.
- Ensure that the distance between the mirror and the scale is 1 m, since the definition of figure of merit demands that.

- Ensure that a damping key is connected to protect the galvanometer. The damping key is connected such that when it is closed, the galvanometer becomes short circuited and the deflection is reduced to zero.
- The circuit is closed only after taking the required resistances in the boxes.
- If 'r' is a copper wire it should have an additional length for connections at the terminals. Refer exp.no.16.
- 'r' is so small that the deflection is within the scale.
- If the sequence key functions well there is no problem of shorting of BG since the circuit is broken when the sequence key is released. So ensure that after releasing the sequence key there is no contact between the key terminals connected to the secondary.
- There may be some variation in the kick with our speed of action.

Observation and tabulation

Value of the standard resistance, r = ……. ohm

<table>
<tr><th rowspan="3">Sl. No.</th><th colspan="6">Mean throw in the galvanometer</th><th rowspan="3">Mean θ mm</th><th colspan="3">Steady deflection 'd' with voltage across 'r'</th><th rowspan="3">$\frac{\theta}{d}$</th></tr>
<tr><th colspan="3">Left</th><th colspan="3">Right</th><th rowspan="2">Left mm</th><th rowspan="2">Right mm</th><th rowspan="2">Mean d mm</th></tr>
<tr><th>θ_1 mm</th><th>θ_3 mm</th><th>$\theta = \theta_1\left(\frac{\theta_1}{\theta_3}\right)^{1/4}$ mm</th><th>θ_1 mm</th><th>θ_3 mm</th><th>$\theta = \theta_1\left(\frac{\theta_1}{\theta_3}\right)^{1/4}$ mm</th></tr>
<tr><td>1</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>2</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>3</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
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<tr><td>5</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>6</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

Mean $\frac{\theta}{d}$ = ……….

<table>
<tr><th colspan="3">Time 't' for 20 free oscillations in seconds</th><th rowspan="2">Period of oscillation T= t/20 seconds</th></tr>
<tr><th>1</th><th>2</th><th>Mean</th></tr>
<tr><td></td><td></td><td></td><td></td></tr>
</table>

Mutual inductance, $M = \left(\frac{Tr}{2\pi}\right)\frac{\theta}{d}$ = = ………. Henry

Result

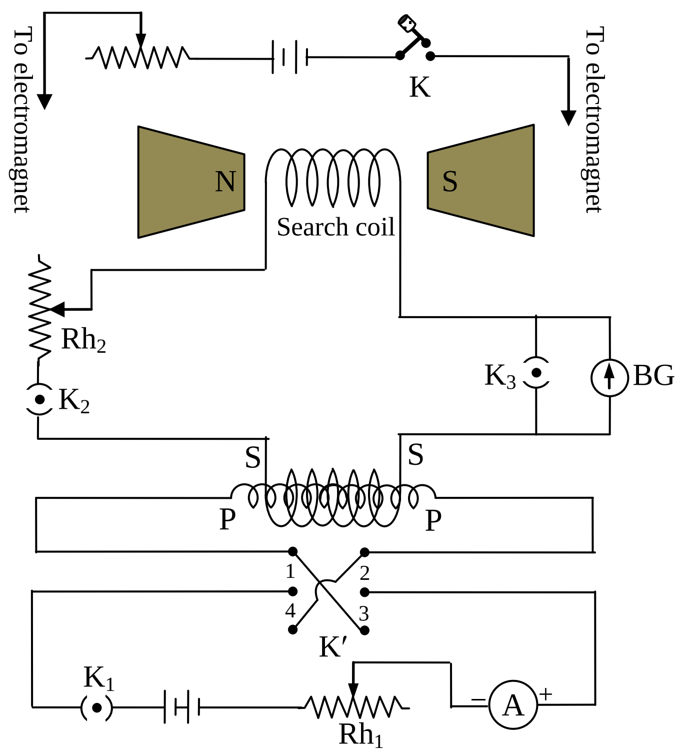
Mutual inductance of the given pair of coils = ……….. henry

Exp.No.2.22

Ballistic galvanometer- Magnetic flux density of an electromagnet using a Search coil

Aim: To determine the magnetic flux density in the region between the pole pieces of an electromagnet using a Search coil.

Apparatus: Ballistic galvanometer, The Search coil, electromagnet, standard solenoid, power supplies, ammeter, two rheostats, three keys, a commutator (current reversing arrangement).



Theory: The Search coil consists of a small coil of N_S turns (about 100) and of area of cross section A_S (about 1 cm^2). It is placed in a magnetic field **B**. Then the magnetic flux linked with the coil is,

$$\phi = N_S B A_S \qquad (1)$$

When the magnetic flux linked with the Search coil is changed, an e m f is induced in the coil and hence a kick is obtained in BG. Let ϕ' be the instantaneous flux linked with the coil. Then the e m f induced in the Search coil is,

$$\varepsilon' = -\frac{d\phi'}{dt} \qquad (2)$$

The instantaneous current through the BG due to this e m f is,

$$I' = \frac{\varepsilon'}{R+G+S} = -\frac{1}{R+G+S}\frac{d\phi'}{dt} \qquad (3)$$

where, R is the resistance in rheostat Rh_2 (or resistance box), G is the galvanometer resistance and S is the resistance of the Search coil. Then the total charge flowing through the BG when the magnetic flux is changed from ϕ to 0,

$$Q' = \int I'dt = -\frac{1}{R+G+S}\int_{\phi}^{0} d\phi' = \frac{\phi}{R+G+S} = \frac{N_S B A_S}{R+G+S} \qquad (4)$$

This charge produces a kick θ in the BG. Then, (by eqn.11b and 13 of the theory of BG)

$$Q' = K\theta\left(1+\frac{\lambda}{2}\right) \qquad (5)$$

where, λ is the damping correction. Thus,

$$\frac{N_S B A_S}{R+G+S} = K\theta\left(1+\frac{\lambda}{2}\right) \qquad (6)$$

To determine K, we use a standard solenoid consisting of n turns per unit length as the primary coil and a secondary coil of total N turns. Since the primary coil is a long solenoid, there is magnetic field only inside the coil and is given by,

$$B = \mu_0 n I'' \qquad (7)$$

where, n is the number of turns per unit length of the primary coil.

The secondary coil is placed in this magnetic field. The instantaneous magnetic flux linked with single turn of the secondary coil is (assuming there is tight coupling and hence the coefficient of coupling is one),

$$\phi'' = BA \qquad (8)$$

where, A is the area of the secondary through which the magnetic field passes. This area A is equal to the area of the secondary if the secondary is well inside the primary and it is equal to the area of the primary if the secondary winding is outside the primary winding.

Let N be the total number of turns of the secondary. Therefore, the flux linked with the secondary coil is,

$$N\phi'' = NAB = \mu_0 N n A I'' \qquad (9)$$

Then by Faraday's law of induction (flux rule),

Induced e m f in the secondary, $\varepsilon'' = -N\frac{d\phi''}{dt}$ (10)

Induced current in the secondary, $I_s = \frac{\varepsilon''}{R+G+S} = -\left(\frac{N}{R+G+S}\right)\frac{d\phi''}{dt}$

Using eqn.9, $= -\left(\frac{\mu_0 N n A}{R+G+S}\right)\frac{dI''}{dt}$ (11)

where, G is the galvanometer resistance and S is the resistance of the secondary.
Total induced charge in the secondary, when the current in the primary coil is reversed,

$$Q' = \int I_s dt = -\left(\frac{\mu_0 N n A}{R+G+S}\right)\int_{I}^{-I} dI'' = \frac{2\mu_0 N n A I}{R+G+S} \qquad (12)$$

Let 'd' be the throw of the spot of light. Then,

$$\frac{2\mu_0 N n A I}{R+G+S} = Kd\left(1+\frac{\lambda}{2}\right) \qquad (13)$$

Dividing eqn.6 by eqn.13 and rearranging, we get,

$$B = \frac{2\mu_0 N n A}{N_S A_S}\left(\frac{I\theta}{d}\right) \qquad (14)$$

Procedure: Connections are made as shown in the figure. The lamp and scale is arranged such that the distance between the mirror and scale is 1 m and the spot of light is stationary at zero line. (The spot of light can be made stationary by closing the damping key). Send a steady current through the electromagnet by pressing the key K and the damping key is made open. Then release the pressing key K. A kick is obtained in B G. If it is out of scale adjust the rheostat Rh_2 (or a suitable resistance there) so that the kick is within the scale.

The primary coil of the standard solenoid is connected to the power source through an ammeter and a special type of commutator key K′. The lever of the key K′ is turned to one side, the gaps 1 and 2 are filled and electrical contact is made there. Now a current flows through the primary of the standard solenoid in one direction. When the lever of K′ is turned to the other side the electrical contact is made in the gaps 3 and 4 and the current through the primary gets reversed.

To begin with the experiment, introduce a suitable resistance in Rh_2. Press the key K and send a current through the electromagnet. The damping key K_3 is made open. Now release the pressing key K. The throw θ in the BG is noted. Repeat the experiment once again and the mean value of θ is determined.

The damping key is closed and the spot of light is brought at the zero position. Then lever of the key K′ is turned to one side. The rheostat Rh_1 is adjusted for a suitable current, say 0.5 ampere. The damping key K_3 is made open and the lever of key K′ is turned to the other side rapidly. The kick 'd' in the B G is noted. Using damping key the spot of light is brought to zero position. Then turn the lever of K to the other side and the kick is again noted and the average 'd' is found out. Repeat the experiment for different values of current in the primary and the kicks are determined in each case. The average value of $\frac{I\theta}{d}$ is determined. The number of turns per unit length 'n' of the primary coil, the total number of turns 'N' of the secondary coil and the total number of turns N_S of the Search coil are noted. The area A of the secondary through which the magnetic field passes is determined by measuring the circumference of secondary or primary as the case may be. The external and the internal diameters of the Search coil are determined. From these, the average diameter and the radius of the Search coil are determined. Then calculate B using eqn.14.

- The change of magnetic flux linked with the Search coil can be achieved either by taking it away from the magnetic field or by changing the current through the electromagnet. In this experiment we release the pressing key K. The current in the electromagnet is reduced to 0 and hence the magnetic flux is reduced from ϕ to 0. Since, in the former case, the induced e m f depends on the speed with which the coil is taken away the kicks are different when the experiment is repeated. The latter case is easy to perform.
- Care must be given to the measurement of A. It is equal to the area of the secondary if the secondary is well inside the primary and it is equal to the area of the primary if the secondary winding is outside the primary winding.
- There may be some variation in the kick with our speed of action.
- Instead of the standard solenoid one can use the H M S to find out the ballistic constant.

Observation and tabulation

Total number of turns of turns of the primary of the standard solenoid, $N' = \ldots\ldots$

Length of the primary of the standard solenoid, $L = \ldots\ldots\text{ cm} = \ldots\ldots\text{m}$

Number of turns per metre of the primary, $n = \frac{N'}{L} = \ldots\ldots$ turns/m

Total number of turns of the secondary of the standard solenoid, $N = \ldots\ldots$

Number of turns of the Search coil, $N_S = \ldots\ldots$

Inner diameter of the Search coil $= \ldots\ldots$ cm

Outer diameter of the Search coil $= \ldots\ldots$ cm

Mean diameter $D = \ldots\ldots$ cm

Radius of the Search coil $r = \frac{D}{2} = \ldots\ldots$ m

Area of the Search coil, $A_S = \pi r^2 = \ldots\ldots\text{ m}^2$

Circumference of primary/secondary , $L' = \ldots\ldots$ cm

* (Strike off which is not applicable)

Radius of the primary/secondary, $r' = \frac{L'}{2\pi} = \ldots\ldots$ m

Area of the secondary through which the magnetic field passes,

A = Area of primary/secondary $= \pi r'^2 = \ldots\ldots\text{ m}^2$

Sl.No.	Throw in BG with Search coil ‘θ’			Current in the primary ampere	Throw in BG when current is reversed ‘d’			$\frac{I\theta}{d}$ ampere
	1 mm	2 mm	Mean mm		Left mm	Right mm	Mean mm	
1								
2								
3								
4								
5								
6								

Mean $\frac{I\theta}{d} = \ldots\ldots$ A

Magnetic field, $B = \frac{2\mu_0 NnA}{N_S A_S}\left(\frac{I\theta}{d}\right) = \qquad = \ldots\ldots$ tesla

Result

Magnetic field in between the pole pieces of the electromagnet, $B = \ldots\ldots$ tesla

*Standard data

Permeability of free space, $\mu_0 = 4\pi\times10^{-7}$ henry/metre

Exp.No.2.23

Ballistic Galvanometer- Absolute capacity of a capacitor

Aim: To determine the capacity of the given condenser using B G.

Apparatus: The ballistic galvanometer, given capacitor, power supply, charge and discharge key, commutator key, ordinary key and three resistance boxes.

Theory: The principle of this experiment is to discharge the charged capacitor through the BG and find out the deflection in BG and from which the capacity is determined. The charge on the capacitor is calculated as follows.

Current through R and S is, $I = \frac{E}{R+S}$

P d across C = P d across R = $IR = \frac{ER}{R+S} = V$

$\therefore$ Charge on C is, $q = CV = \frac{CER}{R+S}$ (1)

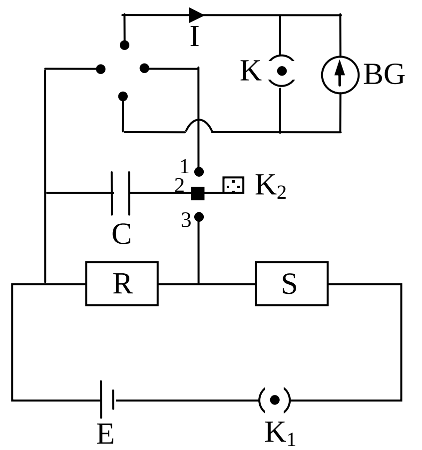


Fig.a

Let ϕ be the deflection produced in the BG when this charged capacitor is allowed to discharge through the BG. Then by eqn.8 of the theory of BG we can write,

$$q = K\phi \quad (2)$$

where, K is the charge sensitivity and $\phi = \phi_1\left(\frac{\phi_1}{\phi_3}\right)^{1/4}$ is the corrected throw in the galvanometer.

But by eqn.15 of the theory,

Charge sensitivity $= \frac{T}{2\pi}\times$current sensitivity

i.e. $K = \frac{Tk}{2\pi}$ (3)

Using eqns.1 and 2, $\frac{q}{\phi} = \frac{Tk}{2\pi} = \frac{CER}{(R+S)\phi}$

$$C = \frac{Tk}{2\pi}\frac{(R+S)\phi}{ER} \quad (4)$$

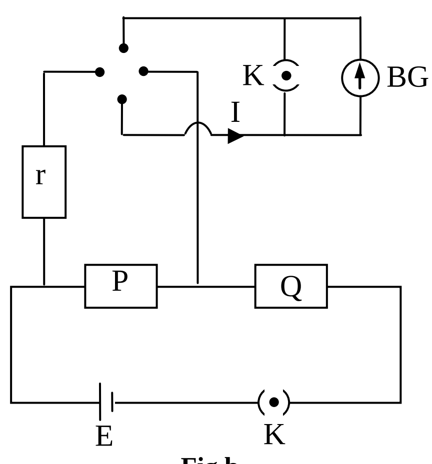


Fig.b

To find out 'k' we follow exp.no.18. Resistances P, Q, R and S are such that,

$$P + Q = R + S = 10{,}000 \text{ ohm} \quad (5)$$

Then by eqn.5 of exp.no.18,

$$k = \frac{E}{(P+Q)G}\left(\frac{P}{\theta}\right) \quad (6)$$

where, θ is the steady deflection in BG when the voltage across 'P' is applied to BG.
Using eqns.4, 5 and 6, we get,

$$C = \frac{T}{2\pi G}\left(\frac{P}{R}\right)\left(\frac{\phi}{\theta}\right) = \frac{T}{2\pi G}\left(\frac{\phi}{R}\right)\left(\frac{P}{\theta}\right) \qquad (7)$$

Procedure

To find out G and $\frac{P}{\theta}$: Connections are made as shown in fig.b. Follow the same procedure as exp.no.18. (Instead of symbol R, r is used here).

To find out $\frac{\phi}{R}$: Connections are made as shown in fig.a. Introduce resistances in R and S such that R + S = 10,000 ohm. R must be sufficiently large (1000 ohm) in this case. This is because we need sufficient potential difference across R so that the capacitor is charged enough to produce an appreciable throw in BG. To start with the experiment press the charge discharge key K_2 so that terminals 2 and 3 are now in contact and thus the capacitor is connected across R. Key K_2 is kept pressed for about 30 seconds and is released. The arrangement of K_2 is such that when it is released it makes contact with terminals 1 and 2 and the capacitor discharges through the galvanometer. This charge produces a throw in the galvanometer. The first throw ϕ_1 and the next throw ϕ_3 of the spot of light on the same side are noted. The corrected throw is calculated as $\phi = \phi_1\left(\frac{\phi_1}{\phi_3}\right)^{1/4}$. Repeat the experiment after reversing the commutator. The entire experiment is repeated for different values of R and the mean value of $\frac{\phi}{R}$ is calculated.

To find out the period of free oscillation of BG the capacitor is again charged and is allowed to discharge through BG. Time for 20 oscillations are found for two times and the period of oscillation is calculated. C is calculated using eqn.7.

Precautions

- Ensure that P + Q = R + S = 10,000 ohm, since the required formula is derived on the assumption that P + Q = R + S = 10,000 ohm.
- Ensure that P is small 1, 2, 3, …….ohm and R is sufficiently large of the order of 1000, 2000, 3000, …….ohm.
- If the capacitance is large, the throw may not be within the scale for R = 1000 Ω, 2000 Ω, etc. In that case try with R = 100 Ω, 200 Ω, 300 Ω, etc.
- Ensure that before discharging the capacitor or sending current to BG the damping key K is made open.
- The e m f of the power supply must be the same for both the cases.

Observation and tabulation

To find out G and $\frac{P}{\theta}$

E m f (open circuit voltage) of the power supply, E = ………… volt

P ohm	Q ohm	Deflection θ in mm to			Resistance G for half deflection			$\left(\frac{P}{\theta}\right)$ ohm/mm
		Left mm	Right mm	Mean mm	Left ohm	Right ohm	Mean Ohm	
							Mean		

To find out $\frac{\phi}{R}$

R ohm	S Ohm	Throw in the galvanometer						Mean ϕ mm	$\frac{\phi}{R}$ mm/ohm
		Left			Right				
		ϕ_1 mm	ϕ_3 mm	$\phi = \phi_1\left(\frac{\phi_1}{\phi_3}\right)^{1/4}$ mm	ϕ_1 mm	ϕ_3 Mm	$\phi = \phi_1\left(\frac{\phi_1}{\phi_3}\right)^{1/4}$ mm		
1000	9000								
2000	8000								
3000	7000								
4000	6000								
5000	5000								

Mean ……….

Determination of period of free oscillation

Time for 20 oscillations second			Period T sec
1	2	Mean	

$$C = \frac{T}{2\pi G}\left(\frac{\phi}{R}\right)\left(\frac{P}{\theta}\right) = ……………. = ………. \text{Farad}$$

Result

Capacity of the given capacitor, C = …………. farad

Exp.No.2.24

Ballistic Galvanometer- High Resistance by Leakage

Aim: To determine the value of the given high resistance by the method of leakage of a capacitor through it.

Apparatus: The ballistic galvanometer, the given high resistance, a capacitor, power supply, two resistance boxes, a charge discharge key, a tap key, two ordinary keys, a commutator key and a small piece of mica sheet.

Theory

Decay of charge in the C-R circuit (Discharging of a capacitor through a resistance)

A capacitor of capacitance C is charged by connecting the switch S to A. Let Q_0 be the initial charge on the capacitor. Now the capacitor is discharged through the resistance R by shifting the switch to B. Let Q be the instantaneous charge on the capacitor. Then the e m f equation of the circuit is,

$$RI + \frac{Q}{C} = 0$$

$$R\frac{dQ}{dt} + \frac{Q}{C} = 0 \qquad (1)$$

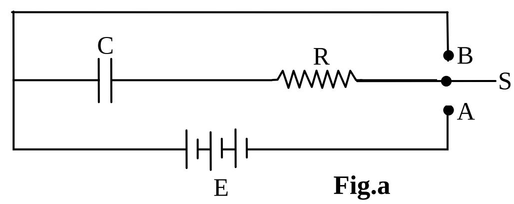


Fig.a

$$R\frac{dQ}{dt} = -\frac{Q}{C}$$

$$\frac{dQ}{Q} = -\frac{1}{CR}dt$$

Integrating we get,

$$\log_e Q = -\frac{1}{CR}t + K \qquad (2)$$

where, K is the constant of integration. When t = 0, $Q = Q_0$.

$$\therefore \quad K = \log_e Q_0 \qquad (3)$$

Substituting eqn.3 in eqn.2 we get,

$$\log_e Q = -\frac{1}{CR}t + \log_e Q_0$$

$$\log_e\left(\frac{Q}{Q_0}\right) = -\frac{1}{CR}t$$

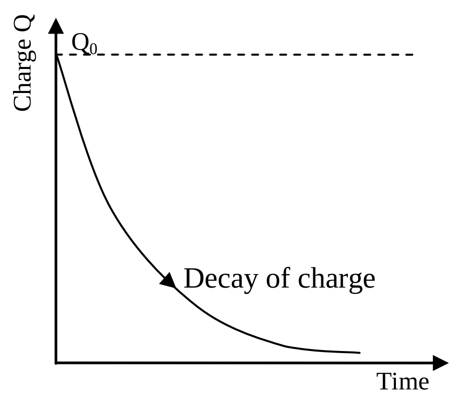


Fig.b

Taking exponential,

$$\frac{Q}{Q_0} = e^{-\frac{t}{CR}}$$

$$\therefore \quad Q = Q_0 e^{-\frac{t}{CR}} \qquad (4)$$

Experimental determination of R: When the capacitor is initially charged to charge Q_0 and is completely discharged through the BG (without leakage through resistance R) we get a throw θ_0 in BG. Then by eqns.8 and 11b of the theory of BG,

$$Q_0 = K\theta_0\left(1+\frac{\lambda}{2}\right) \qquad (5)$$

where, K is the charge sensitivity of the BG and λ is its damping correction.

Now the capacitor is again charged to Q_0. It is then allowed to discharge through the given high resistance for a known time 't' seconds and the remaining charge given by eqn.4 is discharged through the BG. Let θ be the throw in this case. Then,

$$Q = K\theta\left(1+\frac{\lambda}{2}\right) \qquad (6)$$

Using eqns.5 and 6 in eqn.4, we get,

$$K\theta\left(1+\frac{\lambda}{2}\right) = K\theta_0\left(1+\frac{\lambda}{2}\right)e^{-\frac{t}{CR}}$$

$$\frac{\theta_0}{\theta} = e^{\frac{t}{CR}}$$

Taking natural logarithm,

$$\ln\left(\frac{\theta_0}{\theta}\right) = \frac{t}{CR}$$

$$\therefore \qquad R = \frac{t}{C\ln\left(\frac{\theta_0}{\theta}\right)} \qquad (7)$$

Or, using common logarithm,

$$R = \frac{t}{2.303\,C\ln\left(\frac{\theta_0}{\theta}\right)} \qquad (7a)$$

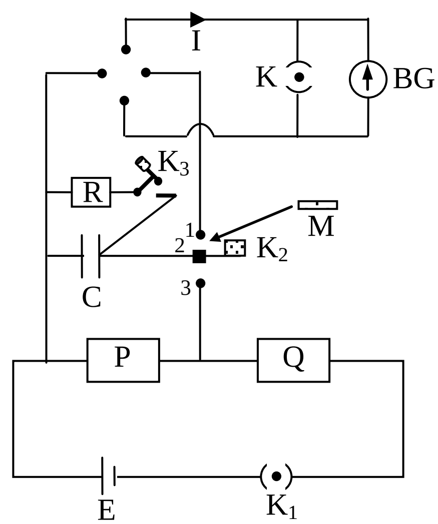


Fig.c

Procedure: Connections are made as shown in fig.c. The high resistance R, whose value is to be determined, is connected across a standard capacitor C through the tap key K_3. K_2 is a charge discharge key. In its normal position terminal 2 is in contact with terminal 1. The lamp and scale is arranged such that the distance between the mirror and scale is 1 m and the spot of light is stationary at zero line. (The spot of light can be made stationary by closing the damping key K). Introduce resistances in P and Q such that P + Q = 10,000 ohm. P must be sufficiently large (1000 ohm) in this case. This is because we need sufficient potential difference across P so that the capacitor is charged enough to produce an appreciable throw in BG.

To begin with the experiment the tap key K_3 is made open and the charge discharge key K_2 is pressed so that the terminals 2 and 3 are in contact with each other. Key K_2 is kept pressed for about 30 seconds so that the capacitor gets charged to Q_0. Then after opening the damping key K the key K_2 is released and thus the terminals 1 and 2 come into contact. Now the capacitor discharges completely through the BG. The throw θ_0 obtained in the B G is noted. The process is repeated by reversing the commutator and the mean value of θ_0 is found.

The capacitor is again charged for the same time interval (30 seconds) to acquire the same initial charge Q_0. Introduce a small non-conducting sheet (mica sheet or a glass strip) M in between the terminals 1 and 2 of the key K_2 and then it is released. Now press the tap key K_3 and simultaneously starts a stop watch. K_3 is kept pressed for 5 seconds and is released. Then the damping key K is made open. Now remove the non-conducting sheet M in between the terminals 1 and 2 and thus allow the capacitor to discharge the remaining charge in it through the BG. The kick in the BG is noted. The process is repeated by reversing the commutator key. Then R is calculated using the eqn.7 or 7a.

The entire experiment may be repeated for different values of ‘t’ and ‘P’ and mean R is found.

- Ensure that during the discharge of the capacitor through the resistance the terminal 2 of key K_2 will not be in contact with terminals 1 and 3.
- There may be some variation in the kick with our speed of action.

Observation and tabulation

Value of the capacitance, C = ………. μF = ………. farad

Trial No.	P ohm	Q Ohm	Throw in the galvanometer						Time of leakage ‘t’ second	R ohm
			Without leakage			With leakage				
			θ_0 left mm	θ_0 right mm	Mean θ_0 mm	θ left mm	θ right mm	Mean θ mm		
1	1000	9000								
2	2000	8000								
3	3000	7000								
4	4000	6000								
5	5000	5000								

Mean R = ……… Ω

Result

The value of the given high resistance, R = ………… ohm